UDC 551.515.2:551.509.314:551.509.57

A NOTE ON DRAWING PROBABILITY SECTORS

PETER P. CHASE

National Hurricane Research Laboratory, ESSA, Miami, Fla.

ABSTRACT

The problem of finding the probability that a tropical storm will be in a given area at forecast time is considered graphically.

With given bivariate data that have a joint normal distribution, a method is presented for plotting ellipses and rays which partition the data area into annuli and sectors having equal values of integrated probability.

Refer to Hoel (1954), page 150. With given data points (x,y) as deviations from the mean, if values in each component are normally distributed with standard deviations σ_x and σ_y and if the components are correlated with coefficient ρ, the probability density function is defined to be:

$$f(x,y)=\frac{\exp\left\{\frac{-1}{2(1-\rho^2)}\left[\frac{x^2}{\sigma_x^2}-2\rho\frac{x}{\sigma_x}\frac{y}{\sigma_y}+\frac{y^2}{\sigma_y^2}\right]\right\}}{2\pi\sigma_x\sigma_y(1-\rho^2)^{1/2}}. \qquad (1)$$

If f is held fixed, the above equation determines an ellipse. Rapp and Isnardi (1951) showed that the inclination angle θ_0 of this ellipse is given by:

$$\theta_0=\frac{1}{2}\arctan\frac{2\rho\sigma_x\sigma_y}{\sigma_x^2-\sigma_y^2}. \qquad (2)$$

They integrated equation (1) over the area of such ellipses, obtaining the probability distribution function P:

$$P=\iint f(x,y)dxdy$$

$$=1-\exp\left\{\frac{-1}{2(1-\rho^2)}\left[\frac{x^2}{\sigma_x^2}-2\rho\frac{x}{\sigma_x}\frac{y}{\sigma_y}+\frac{y^2}{\sigma_y^2}\right]\right\} \qquad (3)$$

where x and y are constrained by $f(x,y)\equiv$constant. The ellipses (1) are more usefully characterized by P:

$$\frac{x^2}{\sigma_x^2}-2\rho\frac{x}{\sigma_x}\frac{y}{\sigma_y}+\frac{y^2}{\sigma_y^2}=-2(1-\rho^2)\ln(1-P). \qquad (4)$$

This integration was accomplished by the transformation:

$$\xi=\left(\frac{x}{\sigma_x}-\frac{\rho y}{\sigma_y}\right)(1-\rho^2)^{-1/2};\quad \eta=\frac{y}{\sigma_y} \qquad (5)$$

which has the property that the ellipses (4) become circles in the ξ, η plane, while rays in the x, y plane remain rays in the ξ,η plane. For polar coordinates r, θ in the x, y plane and s, ϕ in the ξ, η plane, we have:

$$\tan\phi=\frac{\sigma_x(1-\rho^2)^{1/2}\tan\theta}{\sigma_y-\rho\sigma_x\tan\theta} \qquad (6)$$

and

$$\exp(-s^2/2)=1-P. \qquad (7)$$

Integration of (1) over the elliptical sector A of figure 1 is thus equivalent to integration of the transform of (1) divided by the Jacobian of the transformation over the circular sector B of figure 2 in the ξ, η plane.

$$\iint_A f=(\phi_2-\phi_1)(P_2-P_1)/2\pi \qquad (8)$$

where ϕ_1 and ϕ_2 are the transforms of θ_1 and θ_2. This result shows that given an area divided into annuli having equal probability by ellipses (4), the sectors formed by two fixed rays will have equal probability from one annulus to another.

On a mechanical plotter, the ellipses can be drawn using:

$$r=\pm\left[\frac{-2(1-\rho^2)\ln(1-P)}{\frac{\cos^2\theta}{\sigma_x^2}-2\rho\frac{\cos\theta}{\sigma_x}\frac{\sin\theta}{\sigma_y}+\frac{\sin^2\theta}{\sigma_y^2}}\right]^{1/2} \qquad (9)$$

The area can be divided into N sectors, each having equal probability, by rays θ_j, $j=0, 1, \ldots, N-1$, where θ_0 is given by (2) and:

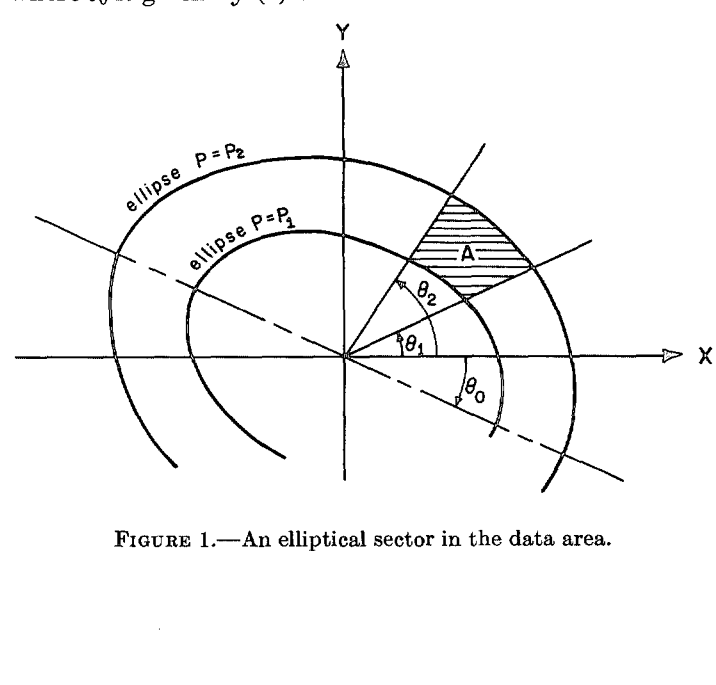


FIGURE 1.—An elliptical sector in the data area.

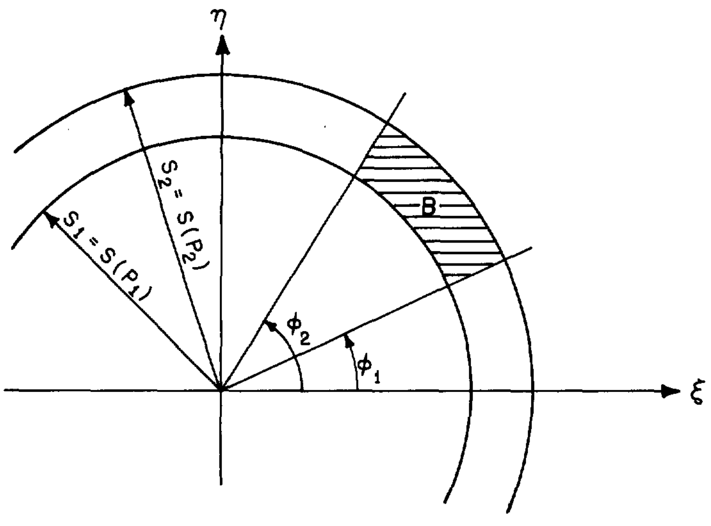
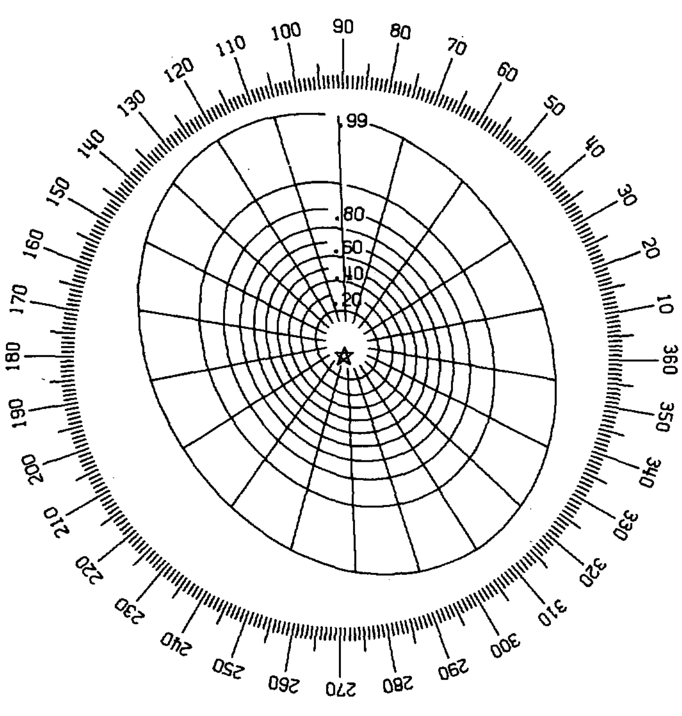


FIGURE 2.—The elliptical sector transformed to ξ, η coordinates.

TABLE 1.—*Statistical parameters generating figure 3*

			Ray	ϕ (deg)	θ (deg)
(y is along forecast track, x is to the right)			1	297	303
$\mu_x = -0.1°$ lat.			2	315	317
$\mu_y = -0.261°$ lat.			3	333	333
$\sigma_x = 1.296°$ lat.			4	351	351
$\sigma_y = 1.416°$ lat.			5	9	11
$\rho = -0.1959$			6	27	33
Latitude=30° N.			7	45	55
Scale=1:10,000,000			8	63	75
			9	81	93
Ellipse	P	s (in.)	10	99	108
1	0.1	0.459	11	117	123
2	0.2	0.668	12	135	137
3	0.3	0.845	13	153	153
4	0.4	1.011	14	171	171
5	0.5	1.177	15	189	191
6	0.6	1.354	16	207	213
7	0.7	1.552	17	225	235
8	0.8	1.794	18	243	255
9	0.9	2.146	19	261	273
10	0.99	3.035	20	279	288



NHC-64 24-HOUR FORECASTS - INDEPENDENT SAMPLE 1962-67
OVERLAY DRAWN FOR 30.0 DEG LAT.
LAMBERT CONFORMAL PROJECTION. 1 TO 10000000
|——| = 1 DEG OF LAT.
PUT STAR ON FCST PSN. TURN DIRECTION OF MOTION TO TOP

FIGURE 3.—Probability sectors and legend drawn by a mechanical plotter.

$$\phi_0 = \arctan \frac{\sigma_x (1-\rho^2)^{1/2} \sin \theta_0}{\sigma_y \cos \theta_0 - \rho \sigma_x \sin \theta_0}, \tag{10}$$

and

$$\phi_j = \phi_0 + 2\pi j/N, \tag{11}$$

$$\theta_j = \arctan \frac{\sigma_y \sin \phi_j}{\sigma_x (1-\rho^2) \cos \phi_j + \rho \sigma_x \sin \phi_j}, \tag{12}$$

for $j = 1, 2, \ldots, N-1$.

Figure 3 was generated on an incremental plotter for 24-hr forecast errors in the NHC-64 hurricane forecast system. (See Miller and Chase, 1966; also Miller, Hill, and Chase, 1968.) Each sector contains 0.5 percent of the total probability. Error components were taken with respect to the forecast direction of motion of a sample of hurricanes between 1962 and 1967; means, standard deviations, and correlation were computed by Mr. Billy M. Lewis of this laboratory. These are given in table 1. The azimuth scale of figure 3 is forecast direction of motion, read at the top. As a transparent overlay, the diagram can be used to estimate the probability of the storm center being within a given area in 24 hr.

REFERENCES

Hoel, P. G., *Introduction to Mathematical Statistics*, John Wiley and Sons, Inc., New York, **1954**, 331 pp.

Miller, B. I., and Chase, P. P., "Prediction of Hurricane Motion by Statistical Methods," *Monthly Weather Review*, Vol. 94, No. 6, June **1966**, pp. 399–406.

Miller, B. I., Hill, E. C., and Chase, P. P., "A Revised Technique for Forecasting Hurricane Movement by Statistical Methods," *Monthly Weather Review*, Vol. 96, No. 8, Aug. **1968**, pp. 540–548.

Rapp, R. R., and Isnardi, A. H., "Variability of Upper Winds," *Progress Report* 138–03, U.S. Army Signal Corps Contract No. DA 36–039 sc–72, Research Division, College of Engineering, New York University, Mar. **1951**, 23 pp.

[*Received February 14, 1969; revised April 9, 1969*]